STRESSES AND STRAINS

1.1 LOAD

Any external force acting on a body is called load. The unit of load are same as that of force. Load is measure in **Newton (N)**

1.2 CLASSIFICATION OF LOAD

1. **According to the effect produced on the body:**

i. **Tensile load:** the load whose effect is to increase the length of the body in the direction of its application is known as tensile load.

ii. **Compressive load:** the load whose effect is to decrease the length of the body in the direction of its application is known as compressive load.

iii. **Shearing load:** the load whose effect is to cause sliding of one face of the body relative to the other is called shearing load.

iv. **Bending load:** the load whose effect is cause a certain degree of curvature or bending in the body is called bending load.

v. **Twisting load:** the effect produced by two couples applied at opposite ends of the body so as to cause one end to rotate about its longitudinal axis relative to the other end are called twisting load.

2. **According to the manner of application of load on the body:**

i. **Dead load:** these load are also known as static load. Magnitude, direction and point of application of these loads are fixed for a given member.

ii. **Live load:** these load are also known as fluctuating load. Magnitude, direction and point of application of these load are not fixed for a given member.

1.3 STRENGTH

The strength of a material may be defined as the maximum resistance which a material can offer to the externally applied load.

1.4 STRESS

Stress may be define as the internal resistance per unit area of cross-section offered by a body against the deformation.

$$\sigma = \frac{F}{A}$$

σ = stress induced in the body

P = load or force acting on a body

A = cross-sectional area of the body

1.5 TYPES OF STRESSES

1. **Direct stress:** when a force is applied perpendicular to the cross-section of the member the stress induced is known as direct stress.

i. **Tensile stress:** when an axial pull is applied on the cross-section area of a body, the stress induced is known as tensile stress.

ii. **Compressive stress:** when an axial push is applied on the cross-sectional area of a body, the stress induced is known as compressive stress.

2. **Shear stress:** when two equal and opposite forces are applied tangentially to the cross section of a body, the stress induced is known as shear stress.

1.6 STRAIN

Strain may be defined as the ratio of change in dimension of the body to the original dimension of the body. Strain is denoted by Epsilon

$$strain = \frac{change\ in\ dimension}{original\ value\ of\ dimension}$$

1.7 types of strains

1. **Tensile strain:** the ratio of increased in length to the original length of the member is termed as tensile strain.
2. **Compressive strain:** the ratio of decrease in length to the original length of the member is termed as compressive strain.
3. **Shear strain:** the ratio of angular deformation to original length along the force is termed as shear strain.
4. **Volumetric strain:** the ratio between the change in volume and the original volume of a member is known as volumetric strain or bulk strain.

1.8 ELASTICITY AND ELASTIC LIMIT

i. **Elasticity:** the deformation produce by external force do not disappear after the removal of external forces, such material are called plastic material

ii. **Elastic limit:** the value of stress corresponding to this limiting force upto which the material is perfectly elastic is known as elastic limit

1.9 HOOKE'S LAW

This law states that when a material is loaded within limit of proportionality, the strain is directly proportional to stress produced by stress

$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$

$$E = \frac{\sigma}{\varepsilon}$$

1.10 LASTIC CONSTANTS

1. **Modulus of elasticity :** It may defined as the ratio of tensile stress and tensile strain or ratio of compressive stress and compressive strain. It is denoted by E.

$$E = \frac{\sigma}{\epsilon}$$

2. **Modulus of rigidity:** the ratio of shear stress and shear strain is known as modulus of rigidity or shear modulus. This is denoted by G

Modulus of rigidity = shear stress/shear strain

3. **Bulk modulus:** when a body is subjected to three mutually perpendicular normal stresses of equal intensity, the ratio of normal stress to the corresponding volumetric strain is known as bulk modulus. It is denoted by K.

K = Normal stress/volumetric strain

1.11 LONGITUDINAL STRAIN AND LATERAL STRAIN

Longitudinal strain: the strain along the direction of the applied force is known as longitudinal strain.

Lateral strain: The strain at right angles to the direction of applied force is known as lateral strain.

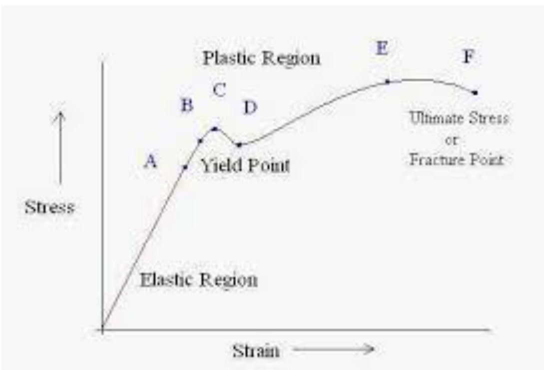
1.12 POISSON'S RATIO

The ratio of lateral strain to the longitudinal strain is known as poisson's ratio. It is denoted by 1/m

Poisson's ratio = lateral strain /longitudinal strain

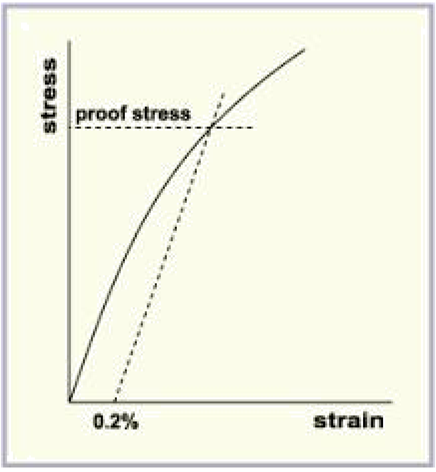
1.13 ENSILE TEST FOR DUCTILE METALS

1. **Limit of proportionality:** limit of proportionality may be defined as that stress at which the stress-strain curve begins to deviate from the straight line.
2. **Elastic limit:** Elastic limit may be defined as the stress developed in the material without any permanent deformation.
3. **Yield point:** the stress corresponding to yield point is known as yield point stress.
4. **Ultimate stress:** the stress attains its maximum value and is known as ultimate stress.
5. **Breaking stress:** the stress corresponding to point F is known as breaking stress.



1.14 PROOF STRESS

Proof stress is the stress necessary to cause a permanent extension equal to a defined percentage of gauge length.



1.15 MAXIMUM OR ULTIMATE TENSILE STRESS

It is defined as the ratio of the maximum load to which a specimen is subjected in a tensile test and the original cross-sectional area of the specimen.

Ultimate stress = maximum load/original cross-sectional area

1.16 WORKING STRESS

The material is not subjected upto ultimate stress, but only upto a fraction of ultimate stress. This stress is known as working stress.

Working stress = ultimate stress/factor of safety

1.17 FACTOR OF SAFETY

The ratio of ultimate stress and working stress is known as factor of safety.

Factor of safety = ultimate stress/working stress

Factor of safety depend upon the following factors:

1. Types of load
2. Frequency of vibration of load
3. Degree of safety required
4. Degree of economy required
5. Dependability of the structure
6. Life span of the structure

1.18 BREAKING STYRESS

It may be defined as the ratio of load at the time of fracture and the original cross-sectional area.

Breaking stress = load at breaking point/original cross sectional

area **1.19 YIELD STRESS**

It is may be define as the lowest stress at which extension of the test specimen takes place without increase in load.

Yield stress = load at yield point/original cross-sectional area

1.20 ECHANICAL PROPERTIES OF MATERIALS

1. **Elasticity/Stiffness**

This is a measure of elastic deformation of a body under stress which is recovered when the stress is released. The ratio of stress to strain in the elastic region is known as stiffness or modulus of elasticity (Young's Modulus). When the stress goes beyond the elastic limit the material will no longer return completely to its original dimension.

2. **Yield (or Proof Strength)**

Stress needed to produce a specified amount of plastic or permanent deformation. (Usually a 0.2 % change in length)

3. **Ultimate Tensile Strength (UTS)**

The maximum stress a material can withstand before fracture.

4. **Ductility**

The amount of plastic deformation that a material can withstand without fracture.

5. **Hardness**

The resistance to abrasion, deformation, scratching or to indentation by another hard body. This property is important for wear resistant applications.

6. **Toughness**

This is commonly associated with impact loading. It is defined as the energy required to fracture a unit volume of material. Generally, the combination of a high UTS and a high ductility results in a higher toughness.

7. **Fatigue Strength and Endurance Limit**

Fatigue failure results from a repeated cyclic application of stress which may be below the yield strength of the material. This is known to be the most common form of mechanical failure of all engineering components. The number of stress cycles needed to cause fatigue failure depends on the magnitude of the stress. Below a certain stress level material does not fail regardless to the number of cycles. This is known as endurance limit and is an important parameter in many design applications.

8. **Creep Resistance**

The plastic deformation of a material which occurs as a function of time when the material is subjected to constant stress below its yield strength. For metals this is associated with high temperature applications but polymers may exhibit creep at low temperatures.

RESILIENCE

2.1 IMPORTANT TERMS

1. **Strain energy:** the work done in straining the body within the elastic limit is known as strain energy

Strain energy = work done

2. **Resilience:** It is a common term used for the total strain energy stored in a body. Sometimes, the resilience may be defined as the capacity of a strained body for doing work on the removal of the straining force.
3. **Proof resilience:** The maximum strain energy which can be stored in a body upto the elastic limit is called proof resilience.
4. **Modulus of resilience:** proof resilience per unit volume the body is known as modulus of resilience.

Modulus of resilience = proof resilience/volume of the body

5. **proof load:** the maximum load which can be applied to a body without its permanent deformation is called proof load.

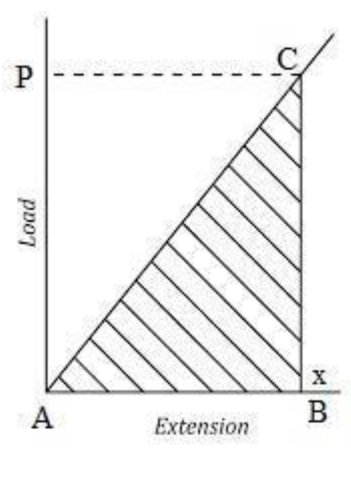
2.2 TYPES OF LOADING

1. Gradually
2. Suddenly
3. With impact

2.3 STRAINED ENERGY STORED IN A BODY DUE TO GRADUALLY APPLIED LOAD

A Gradually applied load is that which is applied gradually on the body i.e. loading begins from zero and increase gradually till the body is fully loaded.

Let us consider a body which is subjected with tensile load which is increasing gradually up to its elastic limit from value 0 to value P and therefore deformation or extension of the body is also increasing from 0 to x and we can see it in following load extension diagram as displayed here.



We have following information from above load extension diagram for body which is subjected with tensile load up to its elastic limit.
σ= Stress developed in the body
E = Young's Modulus of elasticity of the material of the
body A= Cross sectional area of the body
P = Gradually applied load which is increasing gradually up to its elastic limit from value 0 to value P
P=σ.A
x = Deformation or extension of the body which is also increasing from 0 to
x L = Length of the body
V= Volume of the body = L.A
U = Strain energy stored in the body
Let use the value of the extension or deformation "x" in strain energy equation and we will have
U = (1/2) (σ. L/ E).σ. A
U = (1/2) (σ^2/E) L.A
U = (σ^2/2E) V
U = (σ^2/2E) V
Therefore strain energy stored in a body, when load will be applied gradually, will be given by following equation.

$$\frac{\sigma^2}{2E} \text{ x V}$$

$$\text{Proof resilience} = \frac{\sigma^2}{2E} \text{ x V}$$

Modulus of resilience

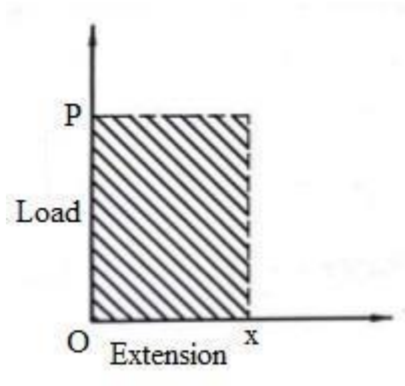
Modulus of resilience = Proof resilience/Volume of the body

$$\text{Modulus of resilience} = \frac{\sigma^2}{2E}$$

2.4 STRAIN ENERGY IN A BODY DUE TO SUDDENLY APPLIED

LOAD A Load applied suddenly on a body is called suddenly applied load.

Let us see the load extension diagram as displayed here for this case where body will be subjected with sudden load and we will find out here the stress induced in the body due to sudden applied load and simultaneously we will also secure the expression for strain energy for this situation.



Let us go ahead step by step for easy understanding, however if there is any issue we can discuss it in comment box which is provided below this post.

We have following information from above load extension diagram for body which is subjected with sudden applied load.

σ = Stress developed in the body due to sudden applied load
E = Young's Modulus of elasticity of the material of the
body A= Cross sectional area of the body
P = Sudden applied load which will be constant throughout the deformation process of the body
x = Deformation or extension of the
body L = Length of the body
V= Volume of the body = L.A
U = Strain energy stored in the body
Strain energy stored in the body = Work done by the load in deforming the
body Strain energy stored in the body = Area of the load extension curve
Strain energy stored in the body = P.
x U = P. x

As we know that maximum strain energy stored in the body U will be provided by the following expression as mentioned here.

$$U = \frac{\sigma^2}{2E} \text{ x } V$$

$$U = \frac{\sigma^2}{2E} \text{ x } A.\ L$$

$$P.\ x = \frac{\sigma^2}{2E} \text{ x } A.\ L$$

Let use the value of the extension or deformation "x" in above equation and we will have

$$P \text{ x } \frac{\sigma}{E} \text{ x } L = \frac{\sigma^2}{2E} \text{ x } A.\ L$$

$$P = \sigma \text{ x } A / 2$$

$$\sigma = 2P/A$$

2.5 STRAIN ENERGY STORED IN A BODY DUE TO IMPACT

LOAD A Load applied with some velocity is called impact load.

Let us see the following figure, where we can see one vertical bar which is fixed at the upper end and there is collar at the lower end of the bar. Let us think that one load is being dropped over the collar of the vertical bar from a height of h as displayed in following figure.

$$P\left(h+\frac{\sigma\ L}{E}\right)=\frac{1}{2}.\frac{\sigma^2}{E}(A\times L)$$

$$Ph+\ \frac{P.\sigma\ L}{E}=\frac{\sigma^2}{2E}(AL)$$

$$\frac{\sigma^2}{2E}-\frac{P.\ \sigma}{A.E}=\frac{P.h}{A.L}$$

$$\sigma^2-\frac{2P\sigma}{A}=\frac{2PhE}{A.L}$$

$$\sigma^2-\frac{2P\sigma}{A}+\frac{P^2}{A^2}=\frac{2PhE}{A.L}+\frac{P^2}{A^2}$$

$$\left(\sigma-\frac{P}{A}\right)^2=\frac{P^2}{A^2}+\frac{2.P.h.E}{A.L}$$

$$\left(\sigma-\frac{P}{A}\right)=\sqrt{\frac{P^2}{A^2}+\frac{2P.h.E}{A.L}}$$

$$\sigma=\frac{P}{A}+\sqrt{\frac{P^2}{A^2}+\frac{2P.h.E}{A.L}}$$

SHEARING FORCE AND BENDING MOMENT

A beam is a structure which is to carry all types of loading coming over it and is economically designed depending upon the type of loading, magnitude of loading and nature of support over which the beam rests.

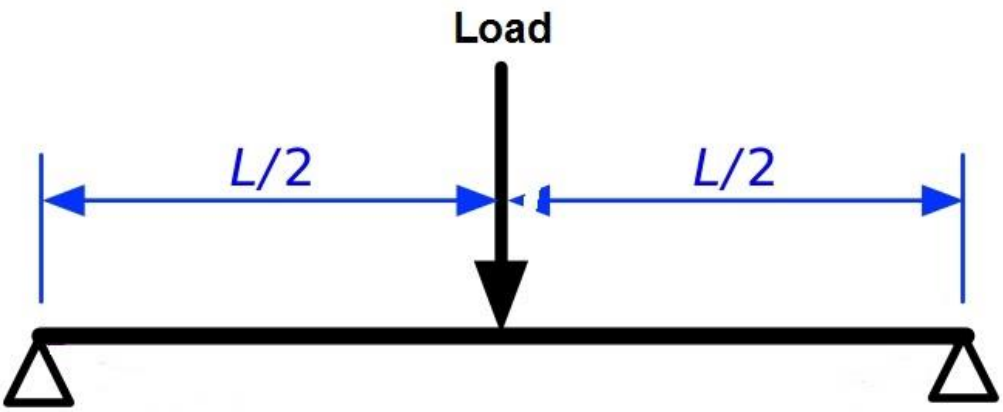
NATURE OF SUPPORT

1. **Hinged Support:** In hinged support position of member is fixed but its direction is not fixed. It can offer resistance to the member horizontally and vertically

Hinged Support

2 **Fixed Support:** In fixed support, both position and direction of member are fixed. Reaction from the support can be any direction.

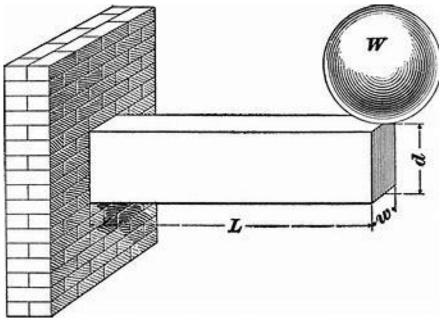
3 **Simply Supported:** In this case member rests freely over the support. The reaction will be normal to support. The position and direction are not fixed. Its horizental component is assumed to be zero.



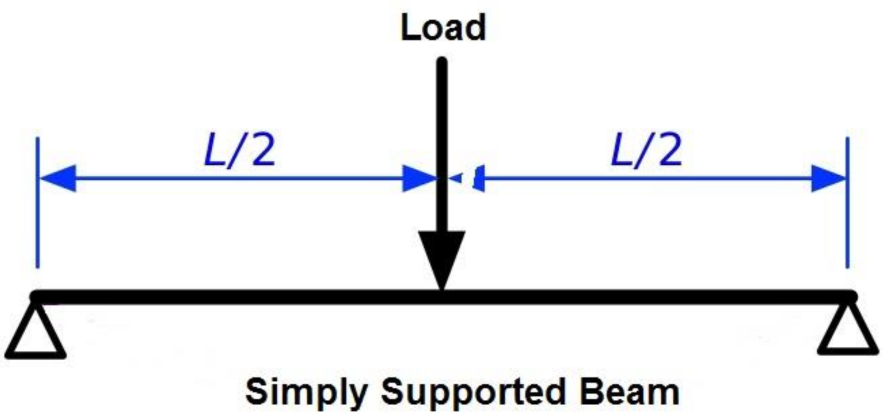
4. **Roller Support:** In this case reaction will always be normal to support and it does not offer any horizental resistance, hence horizental component is always zero.

TYPES OF BEAMS

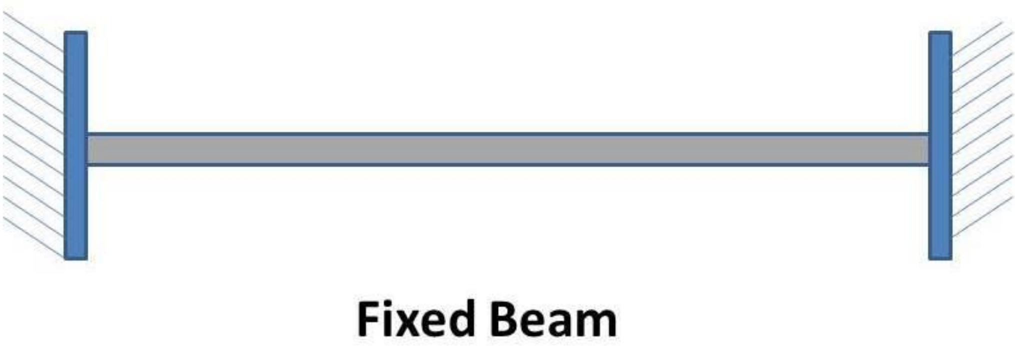
1. **Cantilever:** A beam whose one end is firmly fixed and other end is free, is known as cantilever.



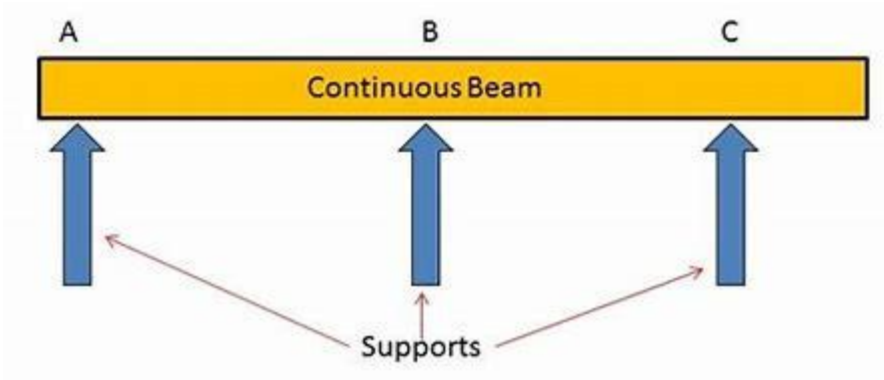
2. **Simply Supported Beam:** A beam whose both ends are simply resting on wall,columns etc. is called simply supported beam.



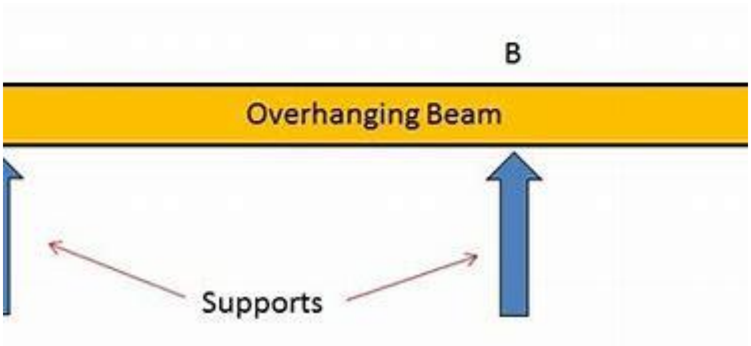
3. **Fixed Beam**: A beam whose both ends are firmly fixed in walls or columns is known as fixed beam.



4 **Continuous Beam:** A beam which has more than two support is known as continuous beam.

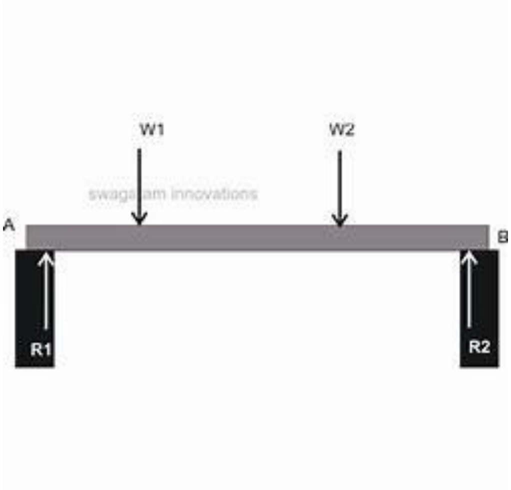


5 **Overhanging Beams:** When the supports are not at the ends of the beam or one or both the ends project beyond the support,then the beam is called overhanging beams.



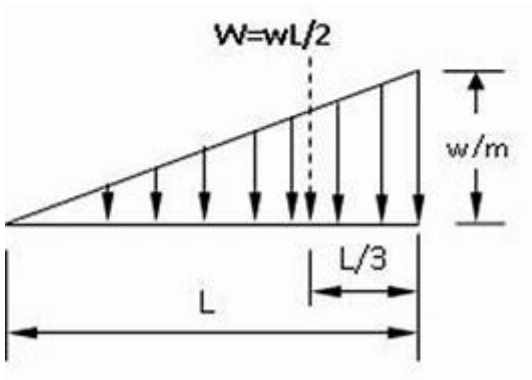
TYPES OF LOADING

1. **Concentrated or point load**: This type of load act on particular point of beam.



2. **Uniformly Distributed Load (U.D.L.):** This type of load is uniformly distributed over whole span of beam on or part of it. It is given in Newton/meter length.

3. **Varying Load:** If the intensity of load distribution increase from zero/unit length to a particular value (say) w/unit length uniformly. It is called varying load.



MOMENT OF INERTIA

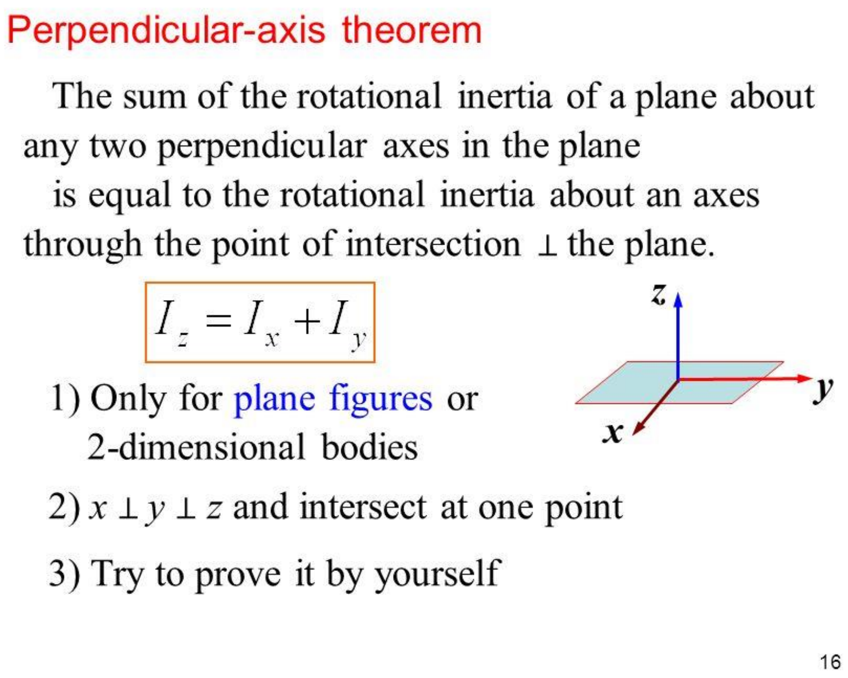
MOMENT OF INERTIA

Moment of inertia (M.I) for a very small area about any axis is given by the product of area and square of distance between centroid of area and the given axis. This is also called second moment of the area.

$$M.I. = Area * (distance)^2$$

THEOREM OF PERPENDICULAR AXIS

The moment of inertia of a plane of lamina about the perpendicular axis to its plane is equal to the sum of moments of inertia of plane of lamina about any two perpendicular axes intersecting each other at the point through which the perpendicular axis passes.

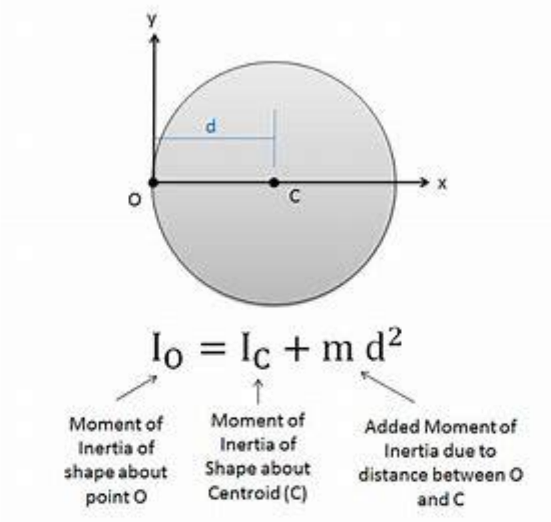


THEOREM OF PARALLEL AXIS

The moment of inertia (M.I.) of a plane of lamina about any axis is equal to its M.I. about a parallel axis passing through its centroid plus the product of the area of lamina and the square of the distance between tow axes.

RADIUS OF GYRATION

The radius of gyration of an area is the distance from the centroid of the area to the



giv en axis. It is denoted by K.

Consider a figure having area (A), moment of inertia (I) and centroid (G)

MOMENT OF INERTIA (M.I.) OF A CIRCULAR DISC

$$Izz = D^4/32.$$

If Ixx = M.I. of disc about X-X axis.

Iyy = M.I. of disc about Y-Y axis.

By the theorem of - axis,

$$Izz = Ixx + Lyy$$

$$Izz = 2Ixx$$

$$Ixx – 1/2Izz = D^4/64$$

$$Iyy = D^4/64$$

MOMENT OF INERTIA (M.I), OF A HOLLOW CIRCULAR DISC

$Izz = /32[D^4-d^4]$
$Ixx = /D^4-d^4]$
$Iyy = /64\ [D4\ -d4]$

BENDING STRESSES

BENDING Equation:-

$$M/I=E/R$$

Hence M/I=E/R=f/y……………………………………BENDING EQUATION

Where

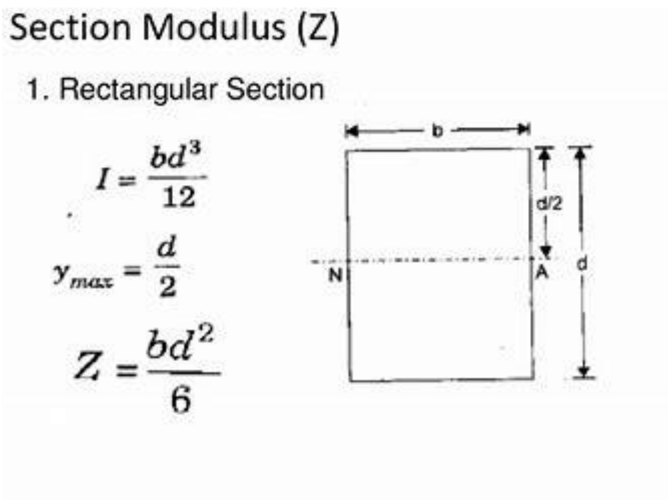
M= B.M. or Moment of resistance of section
I= M.I. of the whole section about N.A.
E=Younge's modulus of elasticity
R=Radius of curvature of N.A.
f= Bending stress at a distance y from N.A.
y=Distance of fibre from N.A.

APPLICATION OF BENDING EQUATION

The bending equation is authentic only for cases where there is pure Bending moment or there is no shearing force. Generally where shearing force is zero, the Bending moment is found to be maximum, in that case Bending equation holds correct results. The Bending Moment at a section accompanied by a shearing force can’t be considered fairly correct for the application of Bending equations.

ASSUMPTIONS IN THE THEORY OF PURE BENDING

1 The material of the beam is homogenous or uniform throughout.
2 The material of the beam is isotropic i.e. having same elastic properties in all the directions.
3 The elastic limit remains within the permissible value.
4 The value of E (Young’s modulus) remains same for tension and compression.
5 The transverse section of the beam remains plane before and after bending.
6 The resultant force on the transverse section of the beam is zero.
7 The beam is assumed to be straight initially.
8 Each layer of beam is free to expand or contract independently of the layers above and below it.
9 The application of load is only in the plane of bending.



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BLENDING STRESSES

INTRODUCTION

When a section of beam is subjected to Bending moment, shear stresses and Bending stresses are set up in the beam. When there is no shearing force it can be considered as pure bending. Theses longitudinal bending stresses can either be compressive or tensile in nature.

BENDING EQUATIONS

M.I. of entire section of beam about N.A.

M/I = E/R.

Hence M/I = E/R = f/y……………………………………

M = B.M. or Moment of resistance of section,
I = M.I of the whole section about N.A.
E = Young’s modulus of elasticity,
R = Radius of curvature of N.A,
F = Bending stress at a distance y from N.A.,
Y = Distance of fibre from N.A.

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6.The resultant Force on the transverse section of the beam is zero.
7.The beam is assumed to be straight initially.
8.Each layer of beam is free to expand or contract independently of the layers above and below it.
9.The application of load is only in the plane of bending.

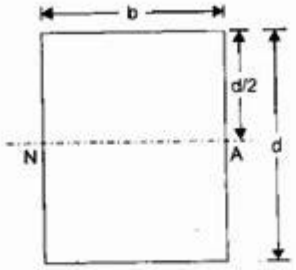
Section Modulus (Z)

1. Rectangular Section

$$I = \frac{bd^3}{12}$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{bd^2}{6}$$



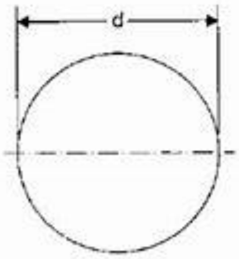
Section Modulus (Z)

3. Circular Section

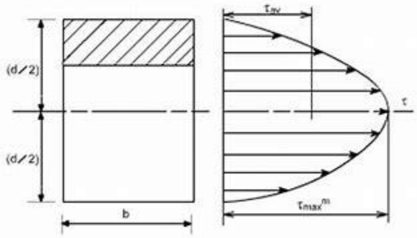
$$I = \frac{\pi}{64}\, d^4$$

$$y_{max} = \frac{d}{2}$$

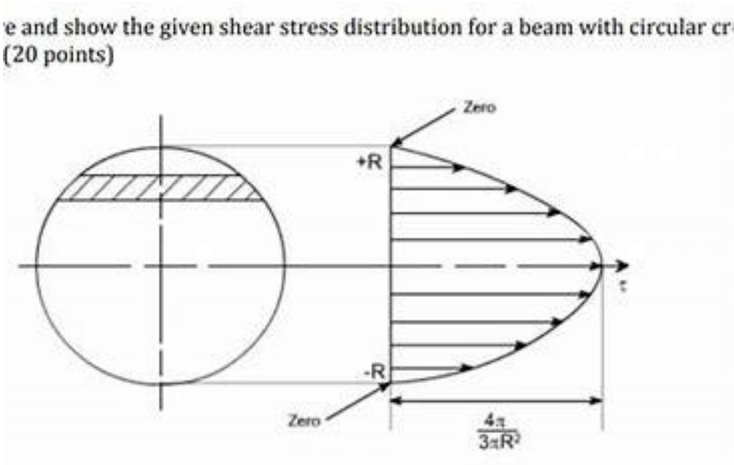
$$Z = \frac{\pi}{32}\, d^3$$



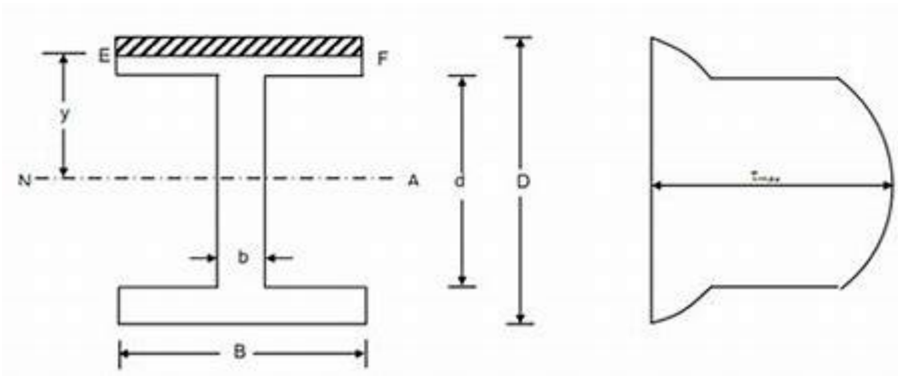
SHEAR STRESSES



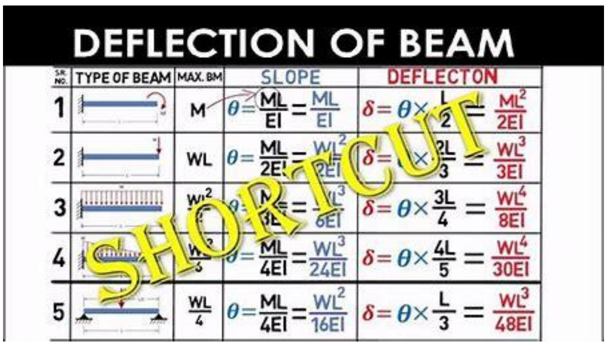
Shear stress is distributed parabolically across the rectangular section
Shear stress will be maximum at y = 0 and will be zero at the extreme ends.



[**shear stressses of I section**](#).



SLOPE AND DEFLECTION


DEFLECTION OF BEAM

SR. NO.	TYPE OF BEAM	MAX. BM	SLOPE	DEFLECTON
1		M	$\theta = \frac{ML}{EI} = \frac{ML}{EI}$	$\delta = \theta \times \frac{L}{2} = \frac{ML^2}{2EI}$
2		WL	$\theta = \frac{ML}{2EI} = \frac{WL^2}{2EI}$	$\delta =$ [illegible] $\times \frac{2L}{3} = \frac{WL^3}{3EI}$
3		[illegible]	[illegible] $= \frac{WL^3}{6EI}$	$\delta = \theta \times \frac{3L}{4} = \frac{WL^4}{8EI}$
4		[illegible]	[illegible] $= \frac{ML}{4EI} = \frac{WL^3}{24EI}$	$\delta = \theta \times \frac{4L}{5} = \frac{WL^4}{30EI}$
5		$\frac{WL}{4}$	$\theta = \frac{ML}{4EI} = \frac{WL^2}{16EI}$	$\delta = \theta \times \frac{L}{3} = \frac{WL^3}{48EI}$